① (A) $$\lim_{n\to+\infty} \frac{2n! + n^2 + 3(n+2)^n}{2^n + n^n} \cdot \frac{2^n}{(1+|\alpha|)^n}$$

$$= \lim_{n\to+\infty} 3\left(\frac{n+2}{n}\right)^n \cdot \left(\frac{2}{(1+|\alpha|)}\right)^n$$

$$= \lim_{n\to+\infty} 3e^2 \left(\frac{2}{1+|\alpha|}\right)^n$$

se $\frac{2}{1+|\alpha|} > 1 \iff 1+|\alpha| < 2 \iff |\alpha| < 1 \iff -1<\alpha<1 \to \boxed{\ell = +\infty}$

se $\boxed{\alpha = \pm 1 \to \ell = 3e^2}$

se $\boxed{|\alpha|>1 \to \ell = 0}$ se $|\alpha| > 1 \iff \boxed{\alpha < -1 \vee \alpha > 1}$

(B) $$\lim_{n\to+\infty} \frac{4n^3 + 2(n+3)^n + n!}{(n+2)^n + 3^n} \cdot \left(\frac{2+|\alpha|}{3}\right)^n$$

$$= \lim_{n\to+\infty} 2\left(\frac{n+3}{n+2}\right)^n \cdot \left(\frac{2+|\alpha|}{3}\right)^n$$

$$= \lim_{n\to+\infty} 2e \left(\frac{2+|\alpha|}{3}\right)^n$$

se $2+|\beta|>3 \iff |\beta| > 1 \iff \alpha<-1 \vee \alpha>1 \Rightarrow \ell = +\infty$

se $\beta = \pm 1 \to \ell = 2e$

se $-1<\beta<1 \Rightarrow \ell = 0$

② ① $\lim_{x\to 0} \dfrac{e^{\operatorname{sen} x} - \cos(x+x^2) - \log(1+x) - 3\operatorname{sen}\left(\frac{x^2}{2}\right)}{(x - \operatorname{tg} x)\left(\cos(x+1) + \log(e+x)\right)}$ ②

$\boxed{N}$

- $\operatorname{sen} x = x - \frac{x^3}{6} + o(x^3)$ $\quad \cancel{\frac{1}{2}x^2} + o(x^3)$

$\Rightarrow$ • $e^{\operatorname{sen} x} = \cancel{1} + \cancel{x} - \cancel{\frac{x^3}{6}} + \frac{1}{2}\left(x^2 - \frac{x^3}{6}\right)^2 + \cancel{\frac{1}{6}x^3} + o(x^3)$

- $-\cos(x+x^2) = -\left[1 - \frac{(x+x^2)^2}{2}\right] + o(x^3) = \cancel{-1} \cancel{+\frac{x^2}{2}} + x^3 + o(x^3)$

- $-\log(1+x) = -\cancel{x} + \cancel{\frac{x^2}{2}} - \frac{x^3}{3} + o(x^3)$

- $-3\operatorname{sen}\left(\frac{x^2}{2}\right) = \cancel{-\frac{3}{2}x^2} + o(x^3)$

$\Rightarrow \quad N = +\frac{x^3}{} - \frac{x^3}{3} + o(x^3) = \frac{2}{3}x^3 + o(x^3)$

$\boxed{D}$: $\quad x - \operatorname{tg} x = -\frac{1}{3}x^3 + o(x^3)$

$\cos(x+1) + \log(e+x) \xrightarrow[x\to 0]{} \cos 1 + 1$

$\Rightarrow \quad \lim_{x\to 0} \frac{N}{D} =$

$$= \lim_{x\to 0} \frac{\frac{2}{3}x^3 + o(x^3)}{-\frac{1}{3}x^3\left(1+\cos(1)\right)} = \boxed{-\frac{2}{1+\cos 1}}$$

$$③\quad \lim_{x\to 0} \frac{\frac{1}{2}\cos\left(x^2+2x\right)+\operatorname{tg}\left(\frac{x^2}{2}\right)-2\operatorname{sen}\left(\frac{x}{2}\right)-\frac{3}{2}+e^{\operatorname{tg}x}}{\left(e^{2x+1}+\log(2+x)\right)\left(\operatorname{sen}x-x\right)} \qquad ③$$

$\boxed{N} =$

$$\bullet\ \frac{1}{2}\cos\left(x^2+2x\right)=\frac{1}{2}\left[1-\frac{\left(x^2+2x\right)^2}{2}+o(x^3)\right]=\frac{1}{2}-\cancel{x^2}-x^3+o(x^3)$$

$$\bullet\ \operatorname{tg}\left(\frac{x^2}{2}\right)=\cancel{\frac{x^2}{2}}+o(x^3)$$

$$\bullet\ -2\operatorname{sen}\left(\frac{x}{2}\right)=-2\frac{x}{2}+\frac{2}{6}\left(\frac{x}{2}\right)^3+o(x^3)=-x+\frac{1}{24}x^3+o(x^3)$$

$$\bullet\ e^{\operatorname{tg}x}=e^{x+\frac{1}{3}x^3+o(x^3)}=1+\left(x+\frac{1}{3}x^3+o(x^3)\right)+\frac{1}{2}\left(x+\frac{1}{3}x^3+o(x^3)\right)^2+\frac{1}{6}x^3+o(x^3)$$

$$=1+\cancel{x}+\frac{1}{3}x^3+\cancel{\frac{x^2}{2}}+\frac{1}{6}x^3+o(x^3)$$

$$\cancel{-\frac{3}{2}}$$

$$N=-x^3+\frac{1}{24}x^3+\frac{1}{3}x^3+\frac{1}{6}x^3+o(x^3)$$

$$=\frac{-24+1+8+4}{24}x^3+o(x^3)$$

$$=\frac{-11}{24}x^3+o(x^3)$$

$$\underline{D}=\operatorname{sen}x-x=-\frac{1}{6}x^3+o(x^3)$$

$$e^{2x+1}+\log(2+x)\xrightarrow[x\to 0]{} e+\log 2$$

$$\Rightarrow \lim_{x\to 0}\frac{N}{D}=\lim_{x\to 0}\frac{-\frac{11}{24}x^3+o(x^3)}{-\frac{1}{6}x^3\left(e+\log 2\right)}=\boxed{+\frac{11}{4(e+\log 2)}}$$

(3) $\displaystyle\int_{e^2}^{e^3} \frac{\log(1+\log x)}{3x\,(\log x-1)^2}\,dx$ (4)

Pongo $t = \log x \rightarrow dt = \frac{1}{x}\,dx$

$$\Rightarrow \int_2^3 \frac{\log(1+t)}{3(t-1)^2}\,dt \overset{\text{per parti}}{=} \left[-\frac{1}{3(t-1)}\log(1+t)\right]_2^3 + \underbrace{\frac{1}{3}\int_2^3 \frac{dt}{(t-1)(t+1)}}_{A}$$

(A) cerco A e B t.c.

$$\frac{1}{(t-1)(t+1)} = \frac{A}{t-1} + \frac{B}{t+1}$$

$$At + A + Bt - B = 1 \iff \begin{cases} A+B=0 \\ A-B=1 \end{cases} \iff \begin{cases} A=-B \\ 2A=1 \end{cases} \begin{cases} B=-\frac{1}{2} \\ A=\frac{1}{2} \end{cases}$$

$$\Rightarrow A = \frac{1}{3}\int_2^3 \left(\frac{1}{2(t-1)} - \frac{1}{2(t+1)}\right)dt$$

$$= \frac{1}{6}\left[\log(t-1) - \log(t+1)\right]_2^3$$

(4) STUDIO DI FUNZIONE (5)

$$f(x) = \log\left(\frac{|x|+1}{|x+1|}\right)$$

DOMINIO

- $D = \begin{cases} \dfrac{|x|+1}{|x+1|} > 0 \Rightarrow \forall x \neq -1 \\ x \neq -1 \end{cases} \Rightarrow D = \mathbb{R} \setminus \{-1\}$

SEGNO e INTERSEZ. CON GLI ASSI

$f(0) = 0$

$f(x) > 0 \iff \dfrac{|x|+1}{|x+1|} > 0$

se $x > 0 \Rightarrow \dfrac{|x|+1}{|x+1|} = \dfrac{x+1}{x+1} = 1$

$\Rightarrow \underline{f(x) = 0 \quad \forall x > 0}$

se $-1 < x < 0 \Rightarrow \dfrac{|x|+1}{|x+1|} = \dfrac{-x+1}{x+1} > 1$ $-x+1 > x+1$, $x<0$ ok

se $x < -1 \Rightarrow \dfrac{|x|+1}{|x+1|} = \dfrac{-x+1}{-x-1} > 1$

$\iff \cancel{-x}+1 > \cancel{-x}-1$ ok

$\Rightarrow f(x) = 0 \quad \forall x \geq 0$

e $f(x) > 0 \quad \forall x < 0,\ x \neq -1$

- LIMITI:

$\lim_{x \to \pm\infty} f(x) = 0$

$\lim_{x \to -1^{\pm}} f(x) = +\infty$

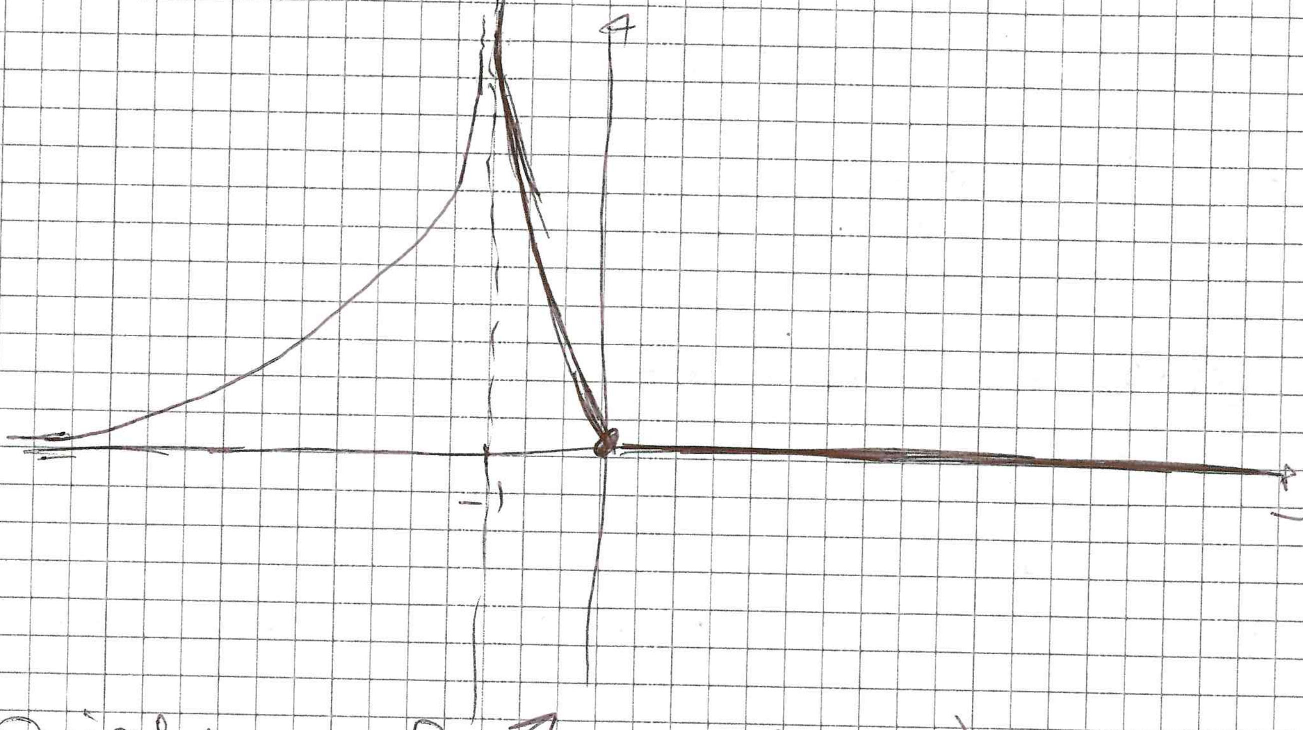
- DERIVATA

$$f'(x) = \frac{|x+1|}{|x|+1}\left(\frac{\operatorname{sgn}(x)|x+1| - \operatorname{sgn}(x+1)(|x|+1)}{|x+1|^2}\right)$$

$\Rightarrow f'(x) > 0$

$\iff \operatorname{sgn}(x)|x+1| - \operatorname{sgn}(x+1)(|x|+1) > 0$

$\iff \begin{cases} -1 < x < 0 \Rightarrow -x-1-(-x+1) = \cancel{-x}-1+\cancel{x}-1 < 0 \Rightarrow f \searrow \\ x < -1 \Rightarrow x+1+(-x+1) = \cancel{x}+1\cancel{-x}+1 > 0 \Rightarrow f \nearrow \end{cases}$



Quindi:

$f \nearrow$ in $(-\infty, -1)$

$f \searrow$ in $(-1, 0)$

$f \equiv 0$ in $[0, +\infty)$

$\nexists \min f$, $\sup f = +\infty$

PTO di NON DERIVAB:

$x = 0$ infatti: $\lim_{x \to 0^+} f'(x) = 0$ (a destra di 0 $f \equiv 0$)

$\lim_{x \to 0^-} f'(x) = \frac{1 \cdot (-1-1)}{1} = -2$